VBAC 2022: Moduli Spaces and Vector Bundles – New Trends

On the occasion of Peter Newstead's 80th birthday

University of Warwick, 25-29 July, 2022

Titles and Abstracts of the Lectures

Usha Bhosle: Poincaré and Picard bundles on the moduli of vector bundles

Abstract: Let Y be an irreducible projective curve with at most nodes as singularities. We present the old as well as new results on the stability of the Poincaré bundles, Picard bundles and projective Picard bundles on the fixed determinant moduli spaces of sheaves on Y. In case Y is smooth, the Picard bundles on the Jacobian have been studied by Fulton, Ein, Lasarsfeld, Kempf (1981-1992). The study of the stability of the Poincare and Picard bundles on the moduli spaces of vector bundles of higher ranks on smooth curves started with a paper by Peter Newstead, V Balaji and Leticia (1997), followed by work of several others.

The results on Jacobians were generalised to compactified Jacobians of nodal curves by myself and A.J. Parameswaran. In the last few years, many results on Picard bundles for higher ranks have been generalised to nodal curves.

Steven Bradlow: Higgs bundles, surface group representations, and branched hyperbolic structures

Abstract: In one of the early triumphs for Higgs bundles Hitchin, in his famous 1987 paper, used rank two Higgs bundles to construct smooth hyperbolic structures on a closed oriented surface and thereby to parameterize Teichmuller space by quadratic differentials. Since then related gauge-theoretic techniques using parabolic structures or vortices have extended constructions of this sort to hyperbolic structures with conic singularities thereby shedding new light on results such as the McOwen-Troyanov existence theorem for such structures. In this talk I will describe an approach to singular hyperbolic structures that revisits Hitchin's original construction and uses only ordinary rank two Higgs bundles. This is based on joint work with Indranil Biswas, Sorin Dumitrescu, and Sebastian Heller.

Arend Bayer: Kuznetsov components of Fano threefolds.

Abstract: I will give a survey of recent progress on Kuznetsov components on Fano threefolds, based on stability conditions and enhanced group actions. In particular, I will explain how these results illuminate the classification and moduli spaces of Fano threefolds.

Leticia Brambilla-Paz: Applications of the Twisted Brill-Noether Theory

Abstract: In this talk I will recall some of the main results of Twisted Brill-Noether theory and give new examples, obtained with Peter Newstead, of non-emptiness of the universal twisted Brill-Noether locus. The non-emptiness of such locus is used to prove non-emptiness of some moduli spaces of unstable bundles and even to describe an open set of the nilpotent cone. As in Brill-Noether theory, such results will depend on the generality of the curve.

Gavril Farkas: The birational geometry of M_g: new developments via non-abelian Brill-Noether theory

Abstract: I will discuss how novel ideas from non-abelian Brill-Noether theory can be used to prove that the moduli space of genus 16 is uniruled and that the moduli space of Prym varieties of genus 13 is of general type. For the much studied question of determining the Kodaira dimension of moduli spaces, both these cases were long understood to be crucial in order to make further progress. I will also explain the use of tropical geometry in order to establish the Strong Maximal Rank Conjecture, necessary to carry out this program.

Carlos Florentino: Topology and MHSs of Representation and Character Varieties of Abelian and Nilpotent groups

Abstract: Let R be the connected component of the identity of the variety of representations of a finitely generated nilpotent group into a connected reductive complex affine algebraic group G. Neither R nor the character variety $R/\!\!/G$ are smooth or projective, in general. Nevertheless, we manage to explicitly determine (via closed and recursive formulae) the Poincaré polynomials and the mixed Hodge structures of R and $R/\!\!/G$. This is the culmination of several works on these varieties over the last 10 years, done in collaboration with S. Lawton, A. Nozad, J. Silva and A. Zamora, and also partly inspired by the article on the E-polynomial of $\mathrm{SL}(2,\mathbf{C})$ character varieties of surface groups, by V. Muñoz, M. Logares and P. Newstead, to whom we dedicate this talk.

Emilio Franco: Degeneration of natural Lagrangians and Prymian integrable systems

Abstract: Starting from an anti-symplectic involution on a K3 surface, one can consider a natural Lagrangian subvariety inside the moduli space of sheaves over the K3. One can also construct a Prymian integrable system following a construction of Markushevich-Tikhomirov, extended by Arbarello-Saccà-Ferretti, Matteini and Sawon-Chen. In this article we address a question of Sawon, showing that these integrable systems and their associated natural Lagrangians degenerate, respectively, into fix loci of involutions considered by Heller-Schaposnik, Garcia-Prada-Wilkins and Basu-Garcia-Prada. Along the way we find interesting results such as the proof that the Donagi-Ein-Lazarsfeled degeneration is a degeneration of symplectic varieties, a generalization of this degeneration, originally described for K3 surfaces, to the case of an arbitrary smooth projective surface, and a description of the behaviour of certain involutions under this degeneration.

Oscar García-Prada: Higgs bundles and symmetric varieties

Abstract: Given a complex reductive Lie group G equipped with an involution with fixed point subgroup H, the usual theory of Higgs bundles is concerned with objects in which the Higgs field takes values in the tangent space of G/H. In this talk, I will explore a theory in which the Higgs field takes values in the symmetric variety G/H itself.

Eloise Hamilton: Stability for unstable Higgs sheaves

Abstract: In this talk I will discuss ongoing work with Joshua Jackson and Victoria Hoskins on the construction of moduli spaces for Higgs sheaves in two different settings: fixing either 1. the Harder-Narasimhan type of the underlying sheaf or 2. the Higgs Harder-Narasimhan type of the Higgs sheaf itself. In both cases the problem can be reduced to that of constructing a quotient of a parameter space by a parabolic group, a difficult problem due to the group being non-reductive. Recent results in Non-Reductive Geometric Invariant Theory due to Jackson and Hoskins suggest the kind of stability condition which should be imposed for Higgs sheaves of a fixed instability type in order to obtain coarse moduli spaces.

Nigel Hitchin: Quadratic maps and rank 2 bundles

Abstract: If E is a very stable rank 2 bundle on a curve C then the determinant defines a quadratic map from the projective space $\mathbf{P}(H^0(C, \mathrm{End}_0 \otimes K))$ to $\mathbf{P}(H^0(C, K^2))$, or equivalently a family of quadrics in $\mathbf{P}^{3g-4}$ parametrized by $\mathbf{P}(H^1(C, K^*))$. The talk will discuss the issue of determining the bundle from this data.

George Hitching: Orthogonal bundles of low rank and quadric fibrations over a curve

Abstract: (Joint work with Insong Choe) Let C be a curve and V an orthogonal vector bundle of rank r over C. For $r \leq 6$, the structure of V can be described using tensor, symmetric and exterior products of bundles of lower rank, essentially due to the existence of exceptional isomorphisms between $\mathrm{Spin}(r, \mathbf{C})$ and other groups for these r. We use this observation to describe moduli spaces of orthogonal bundles for $r \leq 6$. Furthermore, the locus of isotropic vectors in V defines a quadric subfibration of $\mathbf{P}V$. Using classical results on quadrics of low dimension, we exhibit isomorphisms between isotropic Quot schemes of V and certain ordinary Quot schemes of other bundles. In particular, for $r \leq 6$ this gives a method for enumerating the isotropic subbundles of maximal degree of a general V, when there are finitely many.

Victoria Hoskins: Motivic mirror symmetry for Higgs bundles

Abstract: Moduli spaces of Higgs bundles for Langlands dual groups are conjecturally related by a form of mirror symmetry. For SL_n and PGL_n, Hausel and Thaddeus conjectured a topological mirror symmetry given by an equality of (twisted orbifold) Hodge numbers, which was proven by Groechenig–Wyss–Ziegler. We prove that this lifts to an isomorphism of rational Voevodsky motives, and thus in particular an equality of (twisted orbifold) rational Chow groups. Our method is based on Maulik and Shen's approach to the Hausel–Thaddeus conjecture, as well as showing certain motives are abelian, in order to use conservativity of the Betti realisation on abelian motives. This is joint work with Simon Pepin Lehalleur.

Frances Kirwan: Higher quivers, Morse theory and vector bundles over algebraic curves

Abstract: This talk will describe ongoing work with Vidit Nanda on the concepts of higher quivers (especially 2-quivers) and their representations, and of stability conditions on higher quivers in the sense of the work of Haiden-Katzarkov-Kontsevich-Pandit. The aim is to illustrate this using a generalisation of Morse theory and also (in view of the conference's title) moduli of vector bundles over algebraic curves.

Margherita Lelli-Chiesa: Irreducibility of Severi varieties on K3 surfaces

Abstract: Let (S, L) be a general K3 surface of genus g. I will prove that the closure in $|L|$ of the Severi variety parametrizing curves in $|L|$ of geometric genus h is connected for $h \geq 1$ and irreducible for $h \geq 4$, as predicted by a well known conjecture. This is joint work with Andrea Bruno.

Herbert Lange: Decomposition of Jacobians

Abstract: To every smooth projective curve one can associate a principally polarized abelian variety, its Jacobian. The action of a finite group on the curve induces an action on the Jacobian, which gives a decomposition of the Jacobian up to isogeny. In simple cases such a decomposition was already studied at the end of the 19th century in the form of decomposable theta functions.

In the talk I will give a general theorem which works in many cases. The proof relies mainly on representation theory of finite groups. Moreover, several examples will be given where the factors are often Prym varieties. Often the degree of the corresponding isogeny can be computed.

This is joint work with Rubi Rodriguez to be published soon the Springer Lecture Notes of Mathematics.

Marina Logares: E-Polynomials of character varieties of surface groups (I)

Abstract: The study of character varieties has been linked with the study of moduli spaces of vector bundles since the celebrated Narasimhan-Seshadri theorem. Indeed, Prof. Newstead in *Topological properties of some spaces of stable bundles* computed Poincaré polynomials of unitary irreducible representations of the fundamental group of a Riemann surface via studying the topology of the moduli space of stable bundles on a Riemann surface. Since then the study of the topological invariants of G-character varieties have been linked to the study of moduli spaces of G-Higgs bundles through the non-abelian Hodge correspondence. Nevertheless, since the algebro-geometrical invariants of these spaces may be different, in joint work with Peter Newstead and Vicente Muñoz we introduce a method was initiated to study E-polynomials of character varieties.

In the first talk we shall motivate and describe this method, which we call the geometric method. It consists of a description of the matrices appearing in the character variety, a stratification of the character variety, and an analysis of the behavious of E-polynomial under finite quotients and fibrations.

Emanuele Macri: Lagrangian fibrations on hyper-Kähler fourfolds

Abstract: I will present joint work with Olivier Debarre, Daniel Huybrechts and Claire Voisin on the SYZ hyper-Kähler conjecture for fourfolds under certain topological assumptions. As application, this proves a conjecture by O'Grady that a hyper-Kähler fourfold whose cohomology ring is isomorphic to the one of the Hilbert square of a K3 surface is a deformation of a Hilbert square.

Vicente Muñoz: E-Polynomials of character varieties of surface groups (II)

Abstract: In this second talk, we focus on the computation of the E-polynomial of the character variety of the group $G = SL(2, \mathbf{C})$ for an orientable surface of genus g, with a puncture and any holonomy around the puncture. This holonomy can be semisimple but also of Jordan type. We show that there is a recursion on the genus g of the surface, which geometrically amounts to cutting a genus $g + 1$ surface into a genus g and a genus 1 surface. In joint work with Javier Martínez we solve this recursion.

The previous situation leads us to develop a TQFT framework for E-polynomials and motives of the representation varieties of surface groups. We explain this in joint work with Angel González-Prieto and Marina Logares.

Ramanan: Some rationality questions on V B Hyper-elliptic C

Montserrat Teixidor i Bigas: Using reducible curves in the study of vector bundles

Abstract: Reducible curves have been used systematically for at least 40 years in the study of line bundles. They can be equally useful to understand questions about vector bundles of higher rank. We will present some examples as well as applications of the results.

Alfonso Zamora: The moduli stack of principal ρ-sheaves and Gieseker-Harder-Narasimhan filtrations

Abstract: Given a smooth projective variety X and a connected reductive group G defined over a field of characteristic 0, we define a moduli stack of principal ρ-sheaves that compactifies the stack of G-bundles on X. We apply the recent theory of Θ-stratificacions to construct a moduli space of Gieseker semistable principal ρ-sheaves, which provides an intrinsic stack-theoretic construction of the moduli space of semistable principal bundles over higher dimensional varieties. An important outcome is the definition of a Gieseker-Harder-Narasimhan filtration for ρ-sheaves, which induces a stratification of the stack by locally closed substacks and refines the previously known canonical slope parabolic reduction. This is joint work with Andrés Fernández Herrero and Tomás Gómez.